E 7816 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2017

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control/Computer Maintenance and Electronics)

[2013—2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the geometric meaning of the average rate of change of a function $f(x)$ over the interval $[x_1, x_2]$?

2. Find $\lim_{x \to \infty} \left(4 + \frac{1}{x}\right)$.

3. State the derivative product rule.

4. State Rolle's theorem.

5. Define critical point of a function.

6. State first derivative test for monotonic functions.

7. Define the partial derivative of $f(x, y)$ with respect to y at (x_0, y_0).

8. State a formula for implicit differentiation in terms of partial derivatives.

9. Define the hyperbolic cosine of y.

10. Show that $\sin(yi) = i \sinh y$.

$(10 \times 1 = 10)$

Turn over

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Explain why $\lim_{x \to 0} \frac{x}{|x|}$ does not exist ?

12. Find $\lim_{x \to 0} \frac{\sin 3x}{2x}$.

13. Find the fist and second derivatives of $y = \frac{x^3 + 6}{x}$ with respect to x.

14. Find the absolute maximum value of $f(x) = x^2 - 1$ on $[-1, 2]$.

15. Suppose that $f(-2) = 3$ and $f'(x) = 2x$ for all x. Find $f(2)$.

16. Find the value of c of the mean value theorem for the function $f(x) = 1 - x^2$ in $0 \le x \le 2$.

17. If $f(x, y) = (2x - 3y)^3$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

18. Find $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ if $f(x, y) = e^{-2y} \cos 2x$.

19. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$.

20. Show that $8\cos^4 \theta = \cos 4\theta + 4\cos 2\theta + 3$.

21. Show that $\sinh(\alpha + \beta) = \sinh \alpha \cosh \beta + \cosh \alpha + \sinh \beta$.

22. Show that $\sinh^{-1} x = \log\left(x + \sqrt{x^2 + 1}\right)$.

$(8 \times 2 = 16)$

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find $\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2}$.

24. Using the definition of limit prove that $\lim_{x \to 2} x^2 = 4$.

25. Find the slopes of the curve $y^2 - x + 1 = 0$, at the points $(2, -1)$ and $(2, 1)$.

26. If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), prove that there is a point c in (a, b) with $f'(c) = \frac{f(b) - f(a)}{b - a}$.

27. Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and decreasing.

28. Using the definition of partial derivative compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (1, 2) if $f(x, y) = 1 - x + y - 3x^2 y$.

29. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s, y = r + s$.

30. Expand $\sin^3\theta \cos^5\theta$ in a series of sines on multiples of θ.

31. Separate into real and imaginary parts of $\tan^{-1}(\alpha + \beta i)$.

(6 × 4 = 24)

Part D (Long Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Let $f(x) = x + 1, \varepsilon = 0.01$. Find a δ such that for all x with $0 < |x - 4| < \delta$, the equality $|f(x) - 5| < \varepsilon$ holds.

(b) Find the equation of the tangent to the curve $y = x + \frac{2}{x}$ at the point $(1, 3)$.

(c) Find $\frac{d^2y}{dx^2}$ as a function of t if $x = t + \frac{1}{t}, y = t - \frac{1}{t}$.

33. (a) Show that $f(x) = x^4 + 3x + 1$ has exactly one zero in the interval $[-2, -1]$.

(b) The acceleration, initial velocity and initial position of a body moving along a co-ordinate line are respectively

$$a = \frac{d^2s}{dt^2} = 32, \quad v(0) = 20, \quad s(0) = 5.$$

Find the position of the body at time t.

(c) Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing.

34. (a) Find all the second order partial derivatives of $f(x, y) = x^2y + \cos y + y\sin x$.

(b) If $w = 2ye^x - lnz, x = ln(t^2 + 1), y = \tan^{-1} t, z = e^t$, find $\frac{dw}{dt}$ at $t = 1$.

(c) Verify that $f_{xy} = f_{yx}$ if $f(x, y) = xy^2 + x^2y^3 + x^3y^4$.

35. (a) If $\tan(A + iB) = x + iy$, prove that

$$x^2 + y^2 + 2x \cot 2A = 1 \text{ and}$$
$$x^2 + y^2 - 2y \coth 2B = -1.$$

(b) Sum to infinity the series : $\frac{1}{2}\sin\alpha + \frac{1.3}{2.4}\sin 2\alpha + \frac{1.3.5}{2.4.6}\sin 3\alpha + \ldots$

(c) Sum to infinity the series : $\cos\alpha + \frac{c}{\lfloor 1}\cos(\alpha + \beta) + \frac{c^2}{\lfloor 2}\cos(\alpha + 2\beta) + \ldots\ldots$

$(2 \times 15 = 30)$